INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

IFORS

WILEY

INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

Intl. Trans. in Op. Res. 00 (2016) 1–15
DOI: 10.1111/itor.12312

Formulating and solving a radio astronomy antenna connection problem as a generalized cable-trench problem: an empirical study

Kenneth Zyma[a], Julien N. Girard[b], Eric Landquist[c], Gregory Schaper[a] and Francis J. Vasko[c]

[a] *Department of Computer Science and Information Technology, Kutztown University, Kutztown PA 19530, USA*
[b] *AIM/CEA Saclay & LESIA, Observatoire de Paris, UPMC, Université Paris Diderot, CNRS, Paris, France*
[c] *Department of Mathematics, Kutztown University, Kutztown PA 19530, USA*
E-mail: zymakenneth@gmail.com [Zyma]; julien.girard@cea.fr [Girard]; elandqui@kutztown.edu [Landquist]; schaper7532@gmail.com [Schaper]; vasko@kutztown.edu [Vasko]

Received 2 October 2015; received in revised form 8 February 2016; accepted 6 May 2016

Abstract

The low-frequency array, LOFAR, is the world's largest connected low-frequency (LF) radio telescope, combining antenna arrays from across Western Europe into a single telescope. A major extension of an existing LOFAR component in Nançay, France, is currently being developed. The prototype design for this extension consists of a network of 96 LF radio telescope antenna arrays distributed across a 400 × 450 m area. A significant portion of the project's cost involves connecting each antenna array directly to a central control facility via a buried coaxial cable. In his 2013 doctoral dissertation, Girard proposed a cabling layout with the goal of minimizing the total cost to dig the trenches and lay the cables, by modeling the problem as a cable-trench problem (CTP). In this paper, we will discuss Girard's approach and then formulate the problem as a generalized CTP (GCTP) in order to handle physical obstacles more efficiently. To improve upon Girard's solution, we will first apply variations of Prim's algorithm to analyze a number of different GCTPs, based on a variety of cost parameters. Next, we will show that the mixed-integer linear programming formulation of the GCTP finds even better cable and trench layouts using CPLEX. Finally, we will use a new multicommodity flow formulation of the GCTP to find provably optimal solutions using CPLEX. This work has implications beyond the installation in development in Nançay, as any future component of LOFAR can apply the techniques discussed in this paper to significantly reduce construction costs.

Keywords: radio astronomy applications; cable-trench problem; generalized cable-trench problem; mathematical programming formulation

Published by John Wiley & Sons Ltd, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main St, Malden, MA02148, USA.

1. Introduction

Vasko et al. (2002) defined the cable-trench problem (CTP) as a way to combine the shortest path spanning tree problem and the minimum spanning tree problem in a weighted graph with a specified root. The name "cable-trench" comes from the fact that a physical application of the CTP is the problem of minimizing the cost to create a campus network in which each building on a campus is connected to a central server with its own dedicated, underground cable. In Section 2, we will provide graph-theoretic descriptions of the CTP and one of its natural generalizations, the generalized CTP, or GCTP (Vasko et al., 2016). We will then provide mixed-integer linear programming formulations of the CTP and GCTP in Section 3. These formulations can be used to determine optimal solutions of sufficiently small instances of these problems.

Since the seminal CTP paper, there have been a number of publications describing applications of the CTP to model analogous situations. Examples include the application of the CTP to significantly reduce the cost to upgrade and deploy wired and wireless access networks by Nielsen et al. (2008). Marianov et al. (2012) generalized the CTP to forests (disjoint unions of trees) in order to optimize the construction of roads and sawmills for a logging operation and optimize the construction of canals and wells for irrigation. Schwarze (2015) defines the multicommodity CTP in which a network structure is designed such that different cable types (commodities) are inserted into the same trenches. An example of the GCTP is the nontrivial application to vascular image analysis by Jiang et al. (2011) and Vasko et al. (2016).

An interesting logistical problem formulated as a CTP by Girard (2013) is the problem of connecting 96 low-frequency (LF) antenna arrays forming a new radio telescope distributed across a 400×450 m area in the Nançay radio observatory in France to a central control facility via coaxial cables. To protect the cables, trenches will be dug for the cables to run underground. Also, any number of cables can be laid in a given trench. Due to the astrophysical nature of the signal (natural radio emissions) and instrumental constraints of the telescope (analog inputs of the control facility), the use of one analog coaxial cable per array, per polarization is mandatory. In order to preserve the nature and quality of the signal (i.e., no analog to digital conversion, or multiplexing), each of the 96 antenna arrays must be connected directly to the central control facility, which will digitize the signals. A minimum cost (combining both cable and trench costs) configuration will necessarily be a spanning tree of the 96 antenna arrays with the central control facility as the root. For brevity, we refer to this problem as the radio astronomy antenna connection problem (RAACP). In Section 4, we will discuss the RAACP in detail and describe the decomposition approach used by Girard (2013) to generate a feasible solution. We will then show how the GCTP model will make it convenient to handle certain important physical constraints.

In Section 5, we will apply the heuristics developed in Vasko et al. (2016) to find good solutions to the RAACP. Next, we will use a mixed-integer linear programming formulation for the GCTP in order to try to solve the RAACP optimally in CPLEX (within execution time limits). Finally, we will use a new multicommodity formulation of the GCTP (based on the multicommodity formulation of Marianov et al., 2012 for the p-CTP) to find provably optimal solutions using CPLEX. In Section 6, we will make recommendations on particular cable and trench layouts and estimate the cost savings of choosing this layout over the layouts initially proposed in Girard (2013). Finally, we summarize our conclusions in Section 7. We note that this work has more far-reaching implications.

The radio telescope array in Nançay is a component of the current world's largest radio telescope (van Haarlem et al., 2008), the LF array (LOFAR), which combines LF antenna arrays throughout western Europe into a massive scale connected radio telescope. More antenna arrays are planned to be built and incorporated into the LOFAR network, so the methods discussed in this paper can be applied to minimize the cost of further development.

2. Background on the cable-trench and GCTP

2.1. The cable-trench problem

The CTP can be described in graph-theoretic terms as follows. Let $V = \{v_1, \ldots, v_n\}$ be a set of vertices, with $v_1 \in V$ designated at the root vertex, and let $E \subseteq \{(v_i, v_j) : 1 \le i, j \le n, i \ne j\}$ be a set of edges such that the graph $G = (V, E)$ is connected. For each $e \in E$, assign some nonnegative weight $l(e)$ to the edge e to make G a weighted graph. Let τ and γ denote the per unit trench and cable costs, respectively. A solution to the CTP is any spanning tree T of G that minimizes $\tau l_\tau(T) + \gamma l_\gamma(T)$, where $l_\tau(T)$ is the sum of the weights of the edges of T and $l_\gamma(T)$ is the sum of the weights of the paths in T from v_1 to all other $v_k \in V$. Note that in the definition of the CTP, Vasko et al. (2002) did not allow the introduction of Steiner points.

If $\gamma > 0$ and $\tau = 0$, then a solution to the CTP is any shortest path tree of G with root vertex v_1. In contrast, if $\tau > 0$ and $\gamma = 0$, then a solution to the CTP is any minimum spanning tree of G. Thus, solutions to these two limiting cases can be found efficiently, that is, in polynomial time, using Dijkstra's algorithm and Prim's algorithm, respectively.

To connect the definition above to the original application, the root vertex, v_1, represents the central server and the remaining buildings are represented by the other vertices of G. An edge corresponds with an allowable route to dig a trench and lay cables between two buildings. Since each building must be connected by cable to the central server and no auxiliary edges are allowed in the solution spanning tree, the solution to the standard CTP will not be a Steiner tree. In our case, a trench may carry more than one cable once it is dug.

2.2. The generalized cable-trench problem

In the formulation of the GCTP, each edge of the graph is assigned two weights: a cable weight and trench weight. (If both edge weights are the same for each edge, then the GCTP reduces to the CTP.) The obvious motivation for introducing the GCTP is the situation in which the cost to dig a trench is not always proportional to the cost to lay a cable because of obstacles, soil composition, or overhead costs, for example. This formulation will prove very useful to handle practical physical constraints in the RAACP.

Vasko et al. (2002) proved that the CTP is NP-hard. Hence, the GCTP is also NP-hard. Thus, finding optimal solutions to the CTP or GCTP is expected to require exponential time in $n = |V|$ in the worst case. This prohibits one from finding optimal solutions for large examples, such as those encountered in Vasko et al. (2016), which have up to 25,000 vertices and 11 million edges. However, neither optimization software (such as CPLEX®) nor the heuristics in Vasko et al. (2016) have been applied to medium-sized problems, such as the RAACP, where there are 97 vertices and almost 3300 edges.

3. Mathematical programming formulations of the CTP and GCTP

3.1. *A mathematical formulation of the CTP (MFCTP)*

A zero–one mixed-integer linear programming formulation for the CTP, first given in Vasko et al. (2002), is supplied as follows:

$$\text{minimize} \qquad Z = \gamma\left[\sum_{j=1}^{n}\sum_{i=1}^{n} d_{ij}x_{ij}\right] + \tau\left[\sum_{j=1}^{n}\sum_{i=1}^{n} d_{ij}y_{ij}\right] \tag{1}$$

subject to

$$\sum_{j=2}^{n} x_{1j} = n-1 \quad i = 1\ (\text{vertex}1) \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{n} x_{ki} = -1 \quad \text{for all } 2 \le i \le n \tag{3}$$

$$\sum_{j=2}^{n}\sum_{i=1}^{j-1} y_{ij} = n-1 \tag{4}$$

$$(n-1)\, y_{ij} - x_{ij} - x_{ji} \ge 0 \quad \text{for all } i < j \ \ (\text{for all edges}) \tag{5}$$

$$x_{ij} \ge 0 \quad \text{for all } i, j \tag{6}$$

$$y_{ij} \in \{0, 1\} \quad \text{for all } \ i < j, \tag{7}$$

where x_{ij} is the number of cables from $v_i \in V$ to $v_j \in V$ (no cables flow back to v_1, which is the central server); $y_{ij} = 1$ if a trench is dug between v_i and v_j $(i < j)$, and 0 otherwise; and $d_{ij} = l((v_i, v_j))$. The cable cost is γ per unit length and the trench cost is τ per unit length.

Constraint (2) ensures that $n - 1$ cables leave the central facility. Constraint set (3) ensures that each of the $n - 1$ buildings is connected by exactly one cable. Constraint (4) ensures that exactly $n - 1$ trenches are dug. Constraint set (5) ensures that cables are not laid unless a trench is dug. Although the number of cables (constraint set (6)) is only constrained to be nonnegative, these variables will, in fact, be integers because of their relationship to the trench variables which must be either 0 or 1 (constraint set (7)) and the fact that exactly n – 1 of the trench variables are one (constraint (4)).

3.2. *Generalizing the cable-trench problem*

In the GCTP (Vasko et al., 2016), each edge $(v_i, v_j) \in E$ of the graph $G = (V, E)$, has two weights: s_{ij}, the cable weight of (v_i, v_j) and t_{ij}, the trench weight of (v_i, v_j). Then the cable cost from v_i to

v_j is γs_{ij} and the trench cost from v_i to v_j is τt_{ij}. This formulation allows the cable costs to be independent of the trench costs. Setting $d_{ij} = s_{ij} = t_{ij}$ reduces the GCTP to the CTP.

A mathematical formulation for the GCTP (MFGCTP) is identical to MFCTP except for the objective function. Specifically, the objective function for MFGCTP is

$$\text{minimize} \quad Z = \gamma \left[\sum_{j=1}^{n} \sum_{i=1}^{n} s_{ij} x_{ij} \right] + \tau \left[\sum_{j=1}^{n} \sum_{i=1}^{n} t_{ij} y_{ij} \right]. \tag{8}$$

In the next section, we will discuss the RAACP and show how it can be modeled as a GCTP.

4. The radio astronomy antenna array connection problem

4.1. Background

LOFAR is a new and innovative connected radio telescope that officially entered into operation in 2010. The Netherlands Institute for Radio Astronomy (ASTRON) is in charge of operating this network of roughly 50 antenna groups located in the Netherlands, Germany, Great Britain, France, Sweden, and Poland. This system allows for astronomical observations of regions that have been largely unexplored until the present at radio frequencies below 250 MHz using an array of omnidirectional antennas. These antennas are able to monitor the entire sky at all times. After collection, signals from the antennas are amplified in order to transport these signals using coaxial cables to a central station for processing (de Vos et al., 2009). During processing, the analog signals are converted into digital signals where these are used by software to emulate a single conventional antenna that can be digitally pointed anywhere in the visible sky.

On May 20, 2011, the Nançay radio observatory, located in Nançay, France, became the first LOFAR participant in France by hosting one international LOFAR station under the call sign FR606 (Station de Radioastronomie de Nançay, 2011). Currently, Zarka et al. (2012) are working to commission and build the "LOFAR Super Station (LSS)" under the project name NenuFAR (NenuFAR project, 2015). This LSS will consist of 96 LF antenna arrays that will connect to the central station (the LOFAR station back-end) for processing. The placement of antennas has been described for this location in Girard et al. (2012), Girard (2013), and Zarka et al. (2012) and takes into account constraints of the environment at the station, such as existing instruments, buildings, etc. The layout of the antennas and central processing station (labeled "Root") is given in Fig. 1.

Each black dot in Fig. 1 represents an LF antenna array composed of 19 LF antennas arranged in a hexagon that needs to be connected to the central station. The white rectangle is a hard constraint, representing the footprint of the FR606 international LOFAR station already installed on site, which cannot be crossed by a trench directly, that is, the trench must be dug around the obstacle. White circles indicate areas of existing equipment for other projects of the Nançay radio observatory and likewise cannot be crossed directly. All other white areas represent "soft" constraints. These are the areas with an existing buried cable that would have to be exhumed to enable crossings to

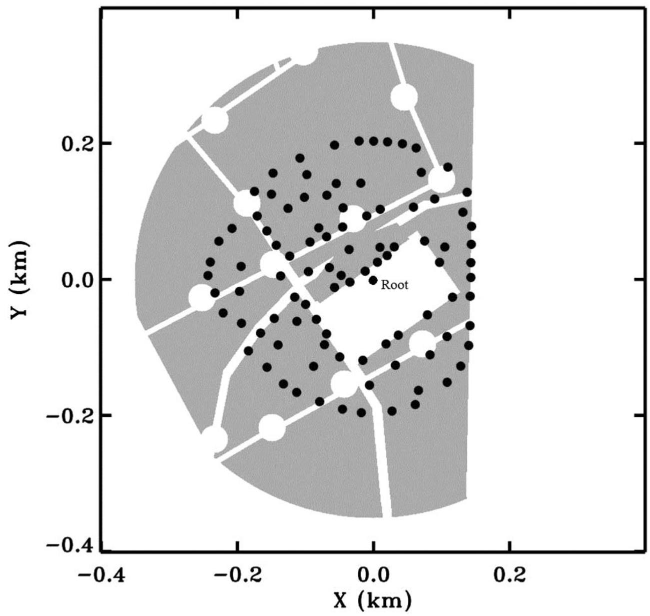

Fig. 1. LSS layout of 96 LF antennas, with the central processing station (Girard, 2013).

the central facility. Thus, a trench may cross any of these areas, but this would incur an unknown additional cost.

Girard (2013) specified a layout for the cable and trench system that connects these 96 antennas to the central station. The cost of cabling this structure using coaxial cables, as well as burying these cables to protect them from physical damage, is nonnegligible due to the size of the layout. Girard (2013) formulated this as a CTP, and his solution is presented in the next section.

4.2. Girard's proposed cable and trench layout

In order to determine a cable and trench layout to connect the 96 antenna arrays to the central station, Girard (2013) decomposed this RAACP into 12 zones (subproblems) indicated in Fig. 2. Each of the 12 subproblems was then solved using a heuristic developed by Vasko et al. (2002), which uses a 1-opt neighborhood search of a given spanning tree to return the spanning tree with the lowest objective function value for all possible values of τ/γ. These zones are then connected using a simple heuristic to minimize the length back to the central station (Girard, 2013). Girard's solution almost minimizes the number of trenches that cross "soft" constraints. There are some zones that are not separated by "soft" constraints, such as zones 8 and 9. Girard reasoned that these should be separated into two zones due to their position relative to the LOFAR station back-end (at $(0,\ 0)$), a "hard" constraint. That is, different cables are to be run to zones 8 and 9. The same assumption is made for zones 11 and 12.

Girard connected the resulting spanning tree in each zone to form one spanning tree of the 96 antenna arrays to the central control facility. His solution is illustrated in Fig. 3. The graph

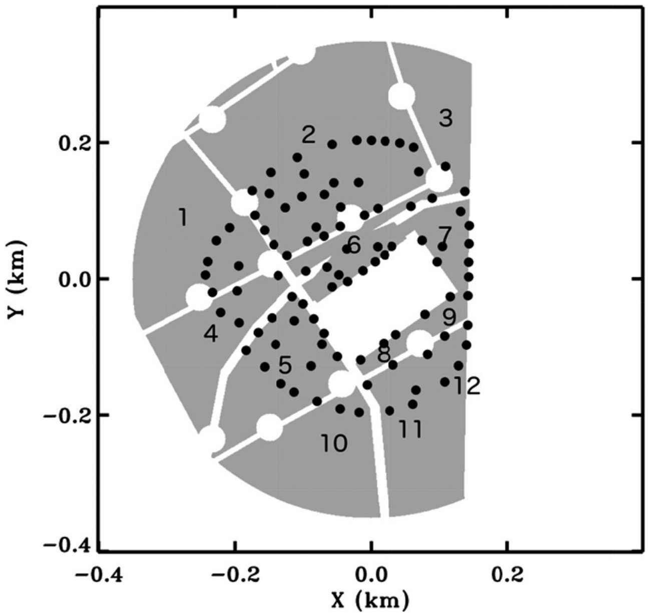

Fig. 2. LSS layout decomposed into 12 zones (Girard, 2013).

representation of Girard's proposed cable and trench layout connecting all 96 antenna arrays to the central control server is illustrated in Fig. 4.

4.3. *The GCTP model of the RAACP*

Consider the antenna array layout in Fig. 1 as a graph with black dots as the 97 vertices and edges connecting any two vertices if the corresponding antenna arrays are within 250 m, the approximate distance from the root vertex to the most distant antenna array. This yields a graph with 3288 edges. If a trench between two arrays does not cross an obstacle, then the cable and trench weights of the corresponding edge are simply the Euclidean distance between the arrays. If the trench between two arrays does not cross a soft obstacle, but must go around a hard obstacle, then the trench and cable weights of the corresponding edge are still identical and correspond to the Euclidean length of the path between the arrays that circumvent the obstacle. If a path must cross a soft obstacle, then the cable weight of the corresponding edge is the Euclidean length of the path, but we will give the edge a greater trench weight. In this case, the trench weight will be the cable weight plus some constant, determined by the distance that a trench digger would ordinarily cover in the time required to carefully dig a trench segment across and in the vicinity of the existing cabling. Thus, this added cost will be represented by some constant Euclidean distance. Hence, accounting for these "soft" constraints in the CTP model establishes this problem as an instance of the GCTP. In the next section, we will estimate the cost of Girard's cable and trench layout and compare it to the costs determined using other solution approaches for a range of parameter values.

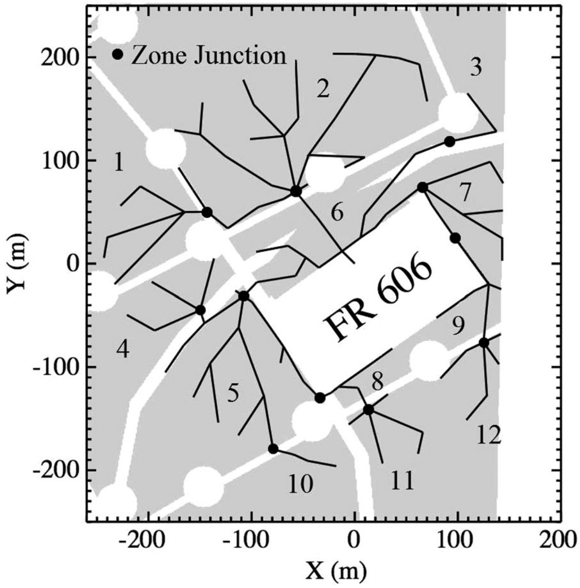

Fig. 3. Proposed cable and trench layout, showing junctions between the zones (Girard, 2013).

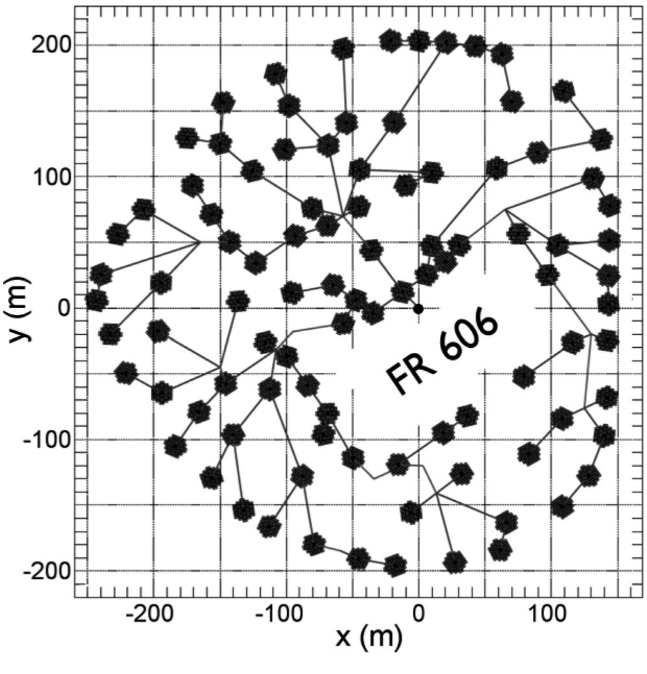

Fig. 4. Proposed cable and trench layout, showing the 96 arrays and the root (Girard, 2013).

Table 1
Total cable-trench costs for Girard's configuration

			"Soft" constraint cost		
γ	τ	τ/γ	0 m	10 m	100 m
2	4	2	53,848	54,248	57,848
4	4	1	93,676	94,076	97,676
4	2	1/2	86,666	86,866	88,666

5. Solution approaches and empirical results

5.1. Testing methodology

Since γ, τ, and the exact cost of crossing a "soft constraint" are unknown, we will use three sets of parameter values. Specifically, the following three sets of γ and τ values are considered here:

(1) the case $\gamma = 2$ and $\tau = 4$ ($\tau/\gamma = 2$),
(2) the case $\gamma = 4$ and $\tau = 4$ ($\tau/\gamma = 1$),
(3) the case $\gamma = 4$ and $\tau = 2$ ($\tau/\gamma = 1/2$),

Note that since both cable and trench costs are significant, $\gamma > 0$ and $\tau > 0$. We will also analyze the impact of three add-on costs for "soft" constraints, namely 0, 10, and 100 m. Hence, with the three cases for γ and τ values described above combined with three "soft" constraint add-on costs, there are a total of nine scenarios that will be analyzed for each solution strategy discussed in this paper. All solution strategies were executed on a MacBook Pro with a 2.4 GHz Intel Core i7 processor, and 8 GB of 1333 MHz DDR3 memory.

5.2. Girard's results

Based on the graph in Fig. 4 and the discussion above, we calculated the total costs of the various scenarios and compared them in Table 1. The total cost is given by $\tau l_\tau(T) + \gamma l_\gamma(T)$, where T is the solution tree in Fig. 4, $l_\tau(T) = 3504$ m, and $l_\gamma(T) = 19914$ m.

5.3. Two heuristic solution procedures for the GCTP

In order to be able to efficiently generate very good solutions to large-scale GCTPs, Vasko et al. (2016) extended a basic greedy algorithm based on Prim's algorithm (MOD_PRIM) that was first proposed by Jiang et al. (2011). Vasko et al. (2016) described two extensions of MOD_PRIM (both multiple pass algorithms): a semigreedy modification called SG_MOD_PRIM and a partially stochastic modification called PS_MOD_PRIM. The best solutions generated by SG_MOD_PRIM and PS_MOD_PRIM are referred to as the BEST_PRIM solution. Details of these algorithms are given in Vasko et al. (2016). With the formulation of the RAACP as a single medium-scale GCTP,

Table 2
BEST_PRIM results with percentage improvements over Table 1

			Total cable and trench costs			Percentage improvement over Table 1		
γ	τ	τ/γ	0 m	10 m	100 m	0 m	10 m	100 m
2	4	2	51,578	52,284	56,856	4.2	3.6	1.7
4	4	1	88,432	89,352	94,932	5.6	5.0	2.8
4	2	1/2	80,876	81,266	84,512	6.7	6.5	4.7

we are able to apply these heuristics and improve on the solutions given in Table 1. The GCTP objective function values for the nine scenarios using BEST_PRIM are given in Table 2, along with percentage improvement over the corresponding values in Table 1. Execution time for each scenario was less than a second.

The minimum improvement (when $\gamma = 2$, $\tau = 4$, and the "soft" constraint cost is 100 m) is 1.7% and maximum improvement (when $\gamma = 4$, $\tau = 2$, and the "soft" constraint cost is 0 m) is 6.7%. The average cost improvement over the nine scenarios is 3.9%. Each of the nine scenarios had a unique spanning tree for its solution.

5.4. *Solving the RAACP as a GCTP using the MFGCTP formulation with CPLEX*

In this section, we present results on applying IBM's optimization software CPLEX to the mixed-integer linear programming (LP) formulation (MFGCTP) of the GCTP model of the RAACP. With this formulation, there are 3288 (binary) variables representing the trench (the y_j) and approximately double the number of cable variables (the x_{ij}). We used all default parameter settings except for Gomory and disjunctive cuts, since a cursory parameter analysis of CPLEX's performance in solving the RAACP as a GCTP using Gomory and disjunctive cuts slowed the convergence of solutions. We considered using the BEST_PRIM solutions as "warm" starts for solving the MFGCTPs in CPLEX, but found that even with "cold" starts, CPLEX very quickly found feasible solutions that were as good as or better than the BEST_PRIM solutions, so we used "cold" starts for all CPLEX results reported.

After an hour of execution time, CPLEX did not terminate for any of the nine scenarios and was not able to guarantee that any solution it had obtained was optimal. Moreover, solution quality had negligible (less than 0.1%) improvement after 10 minutes of execution time. The best solutions that CPLEX obtained after 10 minutes of execution time for the nine scenarios, along with percentage improvements over the BEST_PRIM solutions from Table 2, are given in Table 3. Note that we also included the linear programming (LP) relaxation lower bounds in Table 3. The average gap between the best CPLEX solutions and corresponding LP relaxations is 8.9%.

Table 3 shows that, for a GCTP of the size of the RAACP, even executing CPLEX for 10 minutes resulted in an average cost reduction of 1.3% compared to BEST_PRIM. Since the CPLEX solutions essentially showed no improvement between 10 minutes and one hour of execution time, we were confident that the solutions that we had found were very close to optimal, if not in fact optimal. In

Table 3
The best MFGCTP CPLEX solutions obtained after 10 minutes, with percentage improvements over BEST_PRIM (Table 2)

			Total cable and trench costs LP relaxations			Percentage improvement over Table 2		
γ	τ	τ/γ	0 m	10 m	100 m	0 m	10 m	100 m
2	4	2	50,695	51,444	55,348	1.7	1.6	2.7
4	4	1	87,611	88,428	93,164	0.9	1.0	1.9
4	2	1/2	80,406	80,795	83,762	0.6	0.6	0.9

the next section, we present a new multicommodity flow formulation of the GCTP and results on applying this formulation in CPLEX.

5.5. *Solving the RAACP as a GCTP using the multicommodity formulation with CPLEX*

Marianov et al. (2012) formulated the p-CTP using a multicommodity flow model. Zyma (2015) used this approach to present a novel mathematical programming formulation for the GCTP. Note that the work by Schwarze (2015) on the multicommodity CTP applies multicommodity flows in a different context. There are various cables (and pipes) of differing costs, each carrying a different commodity, such as electricity, water, and natural gas.

In this formulation, each "cable" variable is defined in terms of the vertices, which it is adjacent to, and the terminal vertex for a given cable v_k. Specifically, if $i \neq j,\ k$, then the cable variable x_{ij}^k is 1 if the edge (v_i, v_j) carries a cable with a destination of vertex v_k, and is 0 otherwise. A multicommodity formulation of the GCTP (MCGCTP) is defined as follows:

Minimize

$$Z = \gamma \left[\sum_{k=2}^{n} \sum_{j=2}^{n} \sum_{i=1, i \neq j,k}^{n} s_{ij} x_{ij}^k \right] + \tau \left[\sum_{j=2}^{n} \sum_{i=1}^{j-1} t_{ij} y_{ij} \right], \tag{9}$$

subject to

$$\sum_{j=2}^{n} x_{1j}^k = 1 \quad \text{for all } 2 \leq k \leq n \tag{10}$$

$$\sum_{i=1}^{n} x_{ik}^k = 1 \quad \text{for all } 2 \leq k \leq n \tag{11}$$

$$\sum_{k=2}^{n} \left(x_{ij}^k - x_{ji}^k \right) = 1 \quad \text{for all } 2 \leq i, j \leq n \quad \text{with } i, j \neq k \tag{12}$$

Table 4
Optimal solutions found using CPLEX when the GCTP is formulated as a multicommodity flow problem, with execution times

			"Soft" constraint cost			Execution time (seconds)		
γ	τ	τ/γ	0 m	10 m	100 m	0 m	10 m	100 m
2	4	2	50,695[a]	51,444[a]	55,300	29	35	86
4	4	1	87,611[a]	88,428[a]	93,160	7	12	13
4	2	1/2	80,406[a]	80,795[a]	83,762[a]	5	5	11

[a]The optimal solution was found (but not proven) by the MFGCTP formulation in 10 minutes of execution time or less.

$$\sum_{j=2}^{n}\sum_{i=1}^{j-1} y_{ij} = n-1 \tag{13}$$

$$x_{ij}^k + x_{ji}^k \le y_{ij} \quad \text{for all } 2 \le i, j, k \le n \text{ with } i \ne k \text{ and } i < j \tag{14}$$

$$x_{ij}^k \in \{0, 1\} \quad \text{for all } 1 \le i, j, k \le n, \quad \text{and} \tag{15}$$

$$y_{ij} \in \{0, 1\} \quad \text{for all } 1 \le i < j \le n. \tag{16}$$

Constraints (10) and (11) are the flow balance constraints that ensure for every vertex v_k, one cable leaves the root vertex v_1 and reaches the destination vertex v_k. Constraint (12) enforces that if a cable is laid between two vertices, neither of which is its destination vertex, then that cable must connect to another vertex. This is analogous to Constraint (3) in MFCTP in Section 3.1. Constraint (14) is the "hard" constraint that links cable and trench construction. That is, if $x_{ij}^k = 1$, then y_{ij} must also be 1.

After a cursory parameter analysis of CPLEX's performance in solving these RAACP scenarios using the MCGCTP formulation, we used all default parameter settings except that we turned off presolve, heuristic application, and cuts. With these settings and the MCGCTP formulation, CPLEX terminated with optimal solutions for all nine of the RAACP scenarios. Table 4 shows the results obtained by CPLEX, along with execution times, in seconds.

Table 4 further shows that CPLEX in fact found optimal solutions (but not proven) for seven of the nine problems in 10 minutes or less, using the MFGCTP of the GCTP, and deviated from the optimum by 0.09% (for $\gamma = 2$, $\tau = 4$, and a soft constraint cost of 100 m) and by 0.004% (for $\gamma = 4$, $\tau = 4$, and a soft constraint cost of 100 m) for the other two problems.

6. Discussion

This entire study is based on the very realistic assumption that both cable and trench costs are significant. Given this assumption, if the approximate values for per unit cable cost (γ), per unit trench cost (τ), and the "soft" constraint cost could be determined, then this study has shown that the optimal spanning tree could be determined by using CPLEX and the multicommodity flow formulation of the GCTP.

Table 5
Total cable and trench lengths for optimal spanning trees (in meters)

		"Soft" constraint cost					
		0 m		10 m		100 m	
γ	τ	Cable length	Trench length	Cable length	Trench length	Cable length	Trench length
2	4	18,636	3356	18,692	3335	19,092	3379
4	4	18,376	3577	18,376	3531	18,787	3603
4	2	18,221	3761	18,195	3818	18,399	3883

Table 6
Sensitivity analysis: designed solutions versus actual solutions

		Designed for								
Actual γ, τ		$\gamma = 2,\ \tau = 4$			$\gamma = 4,\ \tau = 4$			$\gamma = 4,\ \tau = 2$		
γ	τ	0	10	100	0	10	100	0	10	100
2	4	50,696	50,724	51,700	50,860	50,996	51,986	51,486	51,662	52,330
4	4	87,968	88,108	89,884	87,612	87,748	89,560	87,928	88,052	89,128
4	2	81,256	81,438	83,126	80,558	80,626	82,354	80,406	80,416	81,362
Average cost		73,307	73,423	74,903	73,010	73,123	74,633	73,273	73,377	74,273
Percent deviation		0.41	0.41	0.85	0	0	0.49	0.36	0.35	0

However, what if we cannot precisely determine the true cable and trench costs in advance? In Table 5, we present the total cable and trench lengths for the optimal spanning trees of the nine scenarios from Table 4. Given this information, we can determine what additional cost would be incurred if we design a trench layout for one cost scenario, when in fact there is a different cost structure.

We will assume that the "soft" constraint cost can be reasonably determined, so we will perform one analysis for each of the three "soft" constraint costs (0, 10, 100). The results are given in Table 6.

Here we see that when the soft constraint is 10 m or less, we should use the solution for the case $\gamma = 4$ and $\tau = 4$. Hence, the average cost deviation would be at most 0.41% greater than the minimal cost. When the soft constraint cost is 100 m, we should use the solution for the case $\gamma = 4$ and $\tau = 2$. Then the average cost deviation would be at most 0.85% from the optimum.

7. Summary and conclusions

In this paper, we discussed an important application of the GCTP, namely the RAACP, which in our example requires the connection of 96 antenna arrays to a central control facility. The GCTP formulation of the RAACP allows the optimization model to account for landscape constraints such as the presence of existing buried cables. Initially, the problem was divided into 12 subproblems and each subproblem was heuristically solved. Next, these 12 solutions were intuitively "connected" into one spanning tree solution of the 96 antennas and the central control facility. A recently developed heuristic, BEST_PRIM, by Vasko et al. (2016), was used to solve the RAACP and showed an average cost reduction of 3.9% over the initial proposed solution. Solutions generated by the execution of the IBM software package CPLEX on the mixed-integer linear programming formulation (given in Vasko et al., 2016) of the GCTP for RAACP resulted in a 1.3% average cost reduction over the BEST_PRIM results after 10 minutes of execution, with negligible improvement thereafter. However, these solutions were not proven to be optimal. Finally, we introduced a new multicommodity flow formulation of the GCTP, which was used to find provable optimal solutions to these GCTP problems with the software CPLEX.

The methods in this paper can immediately be applied in practice to reduce the costs of deploying similar LSS layouts around the other components of the LOFAR telescope. In this paper, we proposed several flexible methods that can be adapted to the specificities of each unique installation site (different soft/hard obstacles) without the need of recasting the full problem. Furthermore, in future, the Square Kilometer Array (SKA) telescope, which will be the next generation of radio telescope, will scale the dimensions of the entire LOFAR up by at least one order of magnitude in size and number of elements. This instrument will be spread out on a continental scale (about 3000 km) and will partly consist of aperture arrays (AA) similar to the LSS, but in a much larger number scale (about 900 AA are planned and each contains about 290 elements, meaning thousands of possible edges). At this scale, smart cost management is mandatory for the feasibility of the project and a 5% improvement on its engineering network implementation (as demonstrated in this paper) can mean millions of dollars in cost reduction.

References

de Vos, M., Gunst, A.W., Nijboer, R., 2009. LOFAR telescope: system architecture and signal processing, *Proc. IEEE, Special Issue on Advances in Radio Telescopes* 97, 8, 1431–1437.

Girard, J.N., 2013. Développement de la Super Station LOFAR et observations planétaires avec LOFAR. PhD thesis, Observatoire de Paris. Available at https://tel.archives-ouvertes.fr/tel-00835834 (accessed 3 June 2015)

Girard, J.N., Zarka, P., Tagger, M., Denis, L., The LSS Team, 2012. Antenna design and distribution of the LOFAR super station. *Comptes Rendus de l'Académie des Sciences – Series IV – Physics-Astrophysics* 13, 1, 33–37.

Jiang, Y., Zhuang, Z., Sinusas, A., Staib, L., Papademetris, X., 2011. Vessel connectivity using Murray's hypothesis. In Fichtinger, G., Martel, A.L., Peters, T.M. (eds) *Proceedings of the MICCAI 2011, LNCS*, Vol. 6893. Springer, Toronto, pp. 528–536.

Marianov, V., Gutiérrez-Jarpa, G., Obreque, C., Cornejo, O., 2012. Lagrangean relaxation metaheuristics for the *p*-cable-trench problem. *Computers & Operations Research* 39, 3, 620–628.

NenuFAR Project, 2015. Available at http://nenufar.obs-nancay.fr/?lang=en (accessed 6 June 2015).

Nielsen, R., Riaz, M., Pedersen, J., Madsen, O., 2008. On the potential of using the cable trench problem in planning of ICT access networks. In Grgic, M. (ed.) *Proceedings of the ELMAR-2008*, IEEE, Zadar, Croatia, pp. 585–588.

Schwarze, S., 2015. The multi-commodity cable trench problem. In *Proceedings of the Twenty-third European Conference on Information Systems*, *ECIS 2015 Completed Research Papers*. AIS, Paper 165, Münster, Germany.

Station de Radioastronomie de Nançay, 2011. Inauguration le vendredi 20 Mai 2011 à Nançay de la station française du réseau européen de radioastronomie LOFAR, plus grand radiotélescope du monde. La France partenaire de ce réseau européen d'antennes inédit (2011/05). Available at http://www.obs-nancay.fr/Inauguration-le-vendredi-20-Mai.html?lang=fr (accessed 3 June 2015).

van Haarlem, M.P., Wise, M.W., Gunst, A.W., 2008. LOFAR: the low-frequency array.*Astronomy and Astrophysics* 556, A2, 1–53.

Vasko, F., Barbieri, R., Rieksts, B., Reitmeyer, K., Stott, K., 2002. The cable trench problem: combining the shortest path and minimum spanning tree problems. *Computers & Operations Research* 29, 5, 441–458.

Vasko, F., Kresge, G., Tal, A., Jiang, Y., Papademetris, X., Landquist, E., 2016. A simple and efficient strategy for solving very large-scale generalized cable-trench problems. *Networks: An International Journal* 67, 3, 199–208.

Zarka, P., Girard, J., Tagger, M., Denis, L., The LSS Team, 2012. LSS/NenuFAR: the LOFAR super station project in Nançay. In Boissier, S., de Laverny, P., Nardetto, N., Samadi, R., Valls-Gabaud, D., Wozniak, H. (eds) *Proceedings of the Annual Meeting of the French Society of Astronomy and Astrophysics*. French Society of Astronomy and Astrophysics, Nice, pp. 687–694.

Zyma, K., 2015. *Solving medium scale instances of the cable-trench problem: applied to the proposed LOFAR super station in Nançay, France*. Master's thesis, Kutztown University, Kutztown, PA.